

# Improved Approaches to the Measurement and Analysis of Torsional Vibration

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## Abstract

*A primary goal of NVH engineering is the identification and control of noise and vibration sources. In recent years the torsional vibration behaviour of engine and powertrain components has gained in significance. This paper discusses several aspects of measuring and analysing torsional vibration and related data. Several torsional vibration measurement techniques are presented, together with remarks on precautions against possible sources of error, and the order of accuracy to be expected of the test results. Two applications requiring multichannel measurement and analysis are outlined.*

## Introduction

Understanding and controlling torsional vibration is of profound importance in vehicle development, refinement and optimisation. The primary source of torsional vibration is the internal combustion engine. Periodic combustion impulses result in rotational speed fluctuations of the crankshaft. Ignition and combustion within a cylinder cause a rapid rise in gas pressure and an angular acceleration of the crankshaft. Gas compression in the next cylinder causes immediate deceleration. Torque pulsations result in crankshaft torsional vibrations which reach the camshaft(s) and auxiliaries via belt or chain drives. In addition, the torsional vibrations which enter the gearbox may be transmitted further via propeller shafts and differentials to the vehicle wheels [1, 2, 3].

Increasing demands to shorten development cycles mean that less time is available for both testing and mathematical modelling. Meaningful solutions may not be arrived at by treating vibration dampers, timing belts, gear stages, etc. as isolated components. Comprehensive studies of the complete system are required which take interactions between individual components into account. Rotec GmbH was formed in 1988 to develop portable, PC-based equipment for use in torsional vibration testing. The company places high emphasis on accurate acquisition of torsional vibration data and the primary analysis methods are based on the revolution domain as opposed

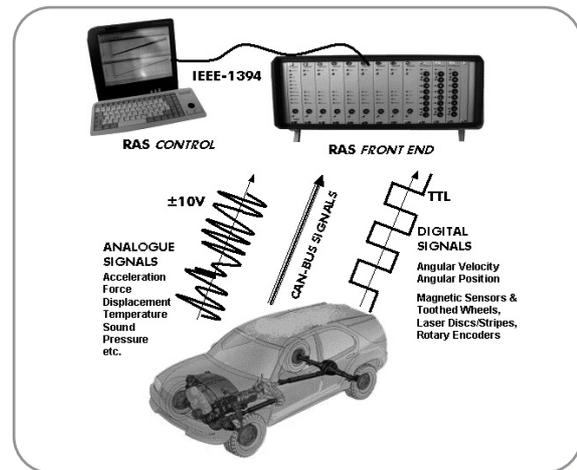


to the time domain. In recent years the company has exploited both the increased computational power of PCs and digital signal processing technology to help make multichannel measurement and analysis quicker and more effective. This paper begins by presenting the new generation of ROTEC-RAS equipment (RAS = Rotation Analysis System). Torsional vibration measurement methods are then reviewed. Sources of error are highlighted and the degree of accuracy of test results is discussed. Two applications in engine and powertrain testing are utilised to illustrate the capabilities of the equipment.

### The New Range of RAS Equipment

Applications requiring multichannel torsional vibration measurement include optimising engine timing and auxiliary drive systems, tuning of vibration dampers, minimising clutch slip, reducing gearbox rattle and transmission error testing of gearsets. This type of testing may also require additional measurement of transverse vibrations synchronous to the rotational data [4, 5]. All RAS channels operate on a common time-base making accurate, phase-matched, cross-channel analyses possible (e.g. accurate calculation of the angular displacement between two rotating shafts). The RAS rotational speed channels require square-wave TTL level signals as input. The time interval between rising (or falling) edges for each pulse period is measured using a high-speed counter/timer (10GHz/40-bit). Input of index pulse and rotational direction signals is also provided for. The RAS analogue channels sample at either 50kHz or 400kHz with 16-bit resolution. Digital downsampling, anti-aliasing protection, programmable gain, AC/DC coupling and differential or ICP inputs are provided. By making use of a commercially available interface board, Controller Area Network (CAN) signals may be directly input and analysed in a similar way to analogue signals. CAN information is increasing in relevance since this is the dominating serial bus system for in-vehicle networks of passen-

ger cars, as well as trucks and buses. RAS systems are modular (both hardware and software), scalable to fit operational needs and used both in test cell and in-vehicle applications. Figure 1 summarises NVH signal types and sources.



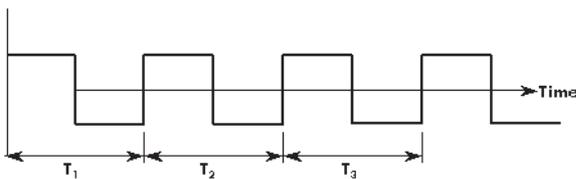
**Figure 1** Test Vehicle, Signals and RAS System

### Angular Velocity Measuring Methods

The digital measurement technique for torsional vibration is based on sampling at equidistant angular intervals around the rotating shaft. This is generally accomplished by one of three methods: (i) mounting an incremental rotary encoder onto the shaft, (ii) scanning a toothed wheel with a magnetic pickup, (iii) targeting reflective/non-reflective (black/white) bar patterns with an optical sensor. The sensor electronics generate an angular velocity signal in the form of a TTL pulse train. The frequency of the pulse train is directly proportional to the angular velocity of the shaft. Rotary encoders with high line counts, as used in single flank testing of gearsets, can provide thousands of pulses per revolution, whereas proximity sensors are limited to a maximum of several hundred pulses per revolution.

Angular sampling provides a fixed number of samples per revolution and is independent of the rota-

tional speed. When time sampling is used, the number of measurement values per revolution varies with rotational speed. A primary advantage of revolution domain analysis is the elimination of leakage errors from spectral components that are order related. Assuming that the angular velocity is constant between adjacent pulses, the instantaneous angular velocity values may be calculated by dividing the actual angular spacing of the physical steps (between gear teeth or encoder lines) by the elapsed time from one positive edge to the next ( $T_1, T_2, T_3, \dots$ ) as shown in Figure 2.



**Figure 2** Standard Rotational-Speed Pulse Train

Figure 2: Standard Rotational-Speed Pulse Train  
The RAS counter/timer frequency of 10GHz per channel (corresponding to a 100ps time base) results in an extremely high angular resolution,  $\Delta\theta$ , which is given by

$$\Delta\theta = \pm\omega \times 10^{-10} \text{ s} \quad (1)$$

with the angular velocity,  $\omega$ , in degrees/sec and  $\Delta\theta$  in degrees. For example, the angular velocity of a shaft rotating at 2400 rpm (40Hz) is 7200 deg/s corresponding to an angular resolution of 1.44  $\mu$ deg.

### Sources of Error in Angular Velocity Measurements

There are three main causes of inaccuracies in angular velocity measurements:

1. Errors in measuring pulse train periodic times
2. Errors due to sensor vibration or relative movement between proximity sensor and target
3. Errors due to variations in tooth spacing

### Measuring Pulse Train Periods

A high number of clock counts per pulse period allows measurement of pulse train periods with an accuracy of  $\pm 1$  count (quantisation error, assuming no clock jitter). Even for encoders with high line counts rotating at high speeds, the RAS 10GHz speed-channel clock ensures a high number of 100psec increments between adjacent pulse train edges. Consider an encoder with 4500 lines rotating at 6000rpm. The pulse frequency is 450kHz (period =  $2.2 \times 10^{-6}$ s) and the number of increments per period is

$$(10\text{GHz} / 450\text{kHz}) = 2.5 \times 10^5 \text{ counter increments}$$

This corresponds to a percentage error of  $4 \times 10^{-6}\%$  and may therefore be neglected. Indeed, the RAS angular resolution – of the order of milli arc seconds at typical rotational speeds – greatly exceeds the measuring uncertainty of even the best rotary encoders available today which are specified to several seconds of arc. Since the RAS time resolution vastly exceeds the accuracy of all angular velocity sensor arrangements, the error associated with measuring pulse train periods is negligible.

### Sensor Movement

Vibration of the proximity sensor in a direction parallel to the target teeth or reflecting pattern produces a signal that is indistinguishable from angular velocity fluctuations. This error can be eliminated by making the sensor mount stiff enough so that sensor vibration is above any angular velocity signal of interest. Relative movement between the target and the proximity sensor during a measurement can have two main causes: The target may move nearer to or further away from the sensor when the shaft is subjected to particular loads. Compared to the normal state, the sensor will encounter a tooth/reflecting marking either too soon or too late and this results in incorrect angular velocity values. A second cause is when toothed wheels or optical targets are mounted eccentrically. A 1st order signal is produced which can-

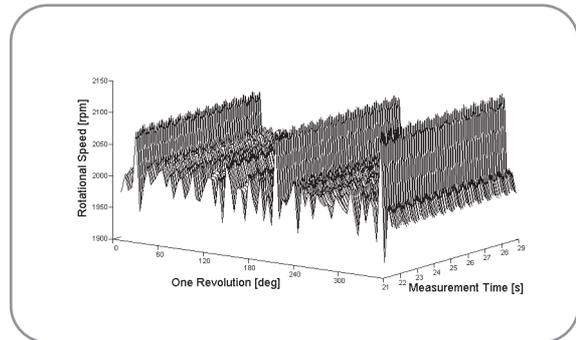


not be distinguished from a genuine 1st order produced by machine vibrations. In order to distinguish between first order machine problems and 1st order imbalance two sensors can be positioned 180 degrees radially opposite to one another. By taking the average value of the two angular velocity signals, the 1st order signal components produced by eccentricity are eliminated.

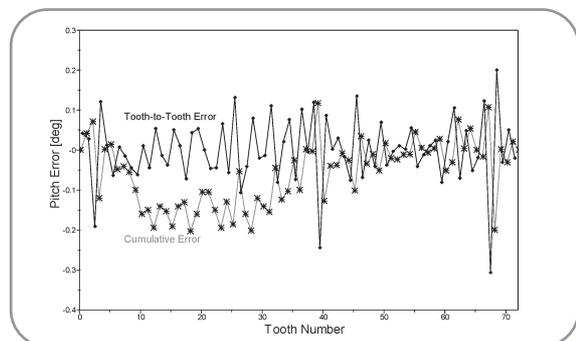
**Tooth Spacing Variation**

All toothed wheels and gears have some degree of variation in tooth spacing. A toothed wheel and magnetic sensor arrangement will generally provide less accurate angular velocity measurement results than a shaft encoder. This section gives an outline of how measurement errors caused by tooth spacing inaccuracy may be quantified.

A rotational speed adapter was machined in the workshop. It comprises an inner toothed wheel with 72 teeth which can be directly coupled to the rotating shaft and a magnetoresistive sensor fitted into an outer bracket which does not rotate. The sensing gap was fixed at 1mm. A calibration measurement over many revolutions was performed by rotating the adapter on a lathe (with minimal speed fluctuation). The sensor signal was input to a RAS speed channel for analysis. Figure 3 clearly shows that the noise pattern in the angular velocity signal primarily caused by tooth spacing variation errors repeats every revolution. Figure 4 shows the actual pitch error from tooth to tooth averaged over many revolutions. Both the individual tooth-to-tooth error and the cumulative error are shown. The cumulative error curve displays a first order inherent in the adapter thought to be caused by the machining process on the milling machine. Once the pattern caused by tooth spacing error has been identified, the data are saved and used to analytically remove the error from subsequent measurements.

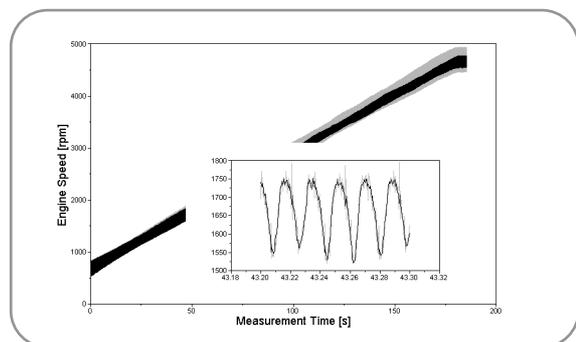


**Figure 3** Pattern of Tooth Spacing Variation



**Figure 4** Tooth-to-Tooth Spacing Errors

Figure 5 shows data from a speed ramp measurement on a 4-cylinder engine using the toothed-wheel adapter bolted onto the front end of the crankshaft. The grey curve represents the raw time history data. The black curve results when the data of Figure 4 are used to remove the tooth spacing error.



**Figure 5** Removal of Tooth Spacing Errors



### Errors in Spectral Analysis Arising from Inadequate Sampling Rates

Shannon's sampling theorem states that a signal may be detected up to half the sampling frequency (the Nyquist frequency). This value of half the sampling frequency is equal to the maximum value of frequency (or order) seen in the spectrum. (An order is a harmonic vibration whose period is an integer number of revolutions of a reference speed channel). The effect of 'aliasing' is to allow frequencies higher than the Nyquist frequency being reflected back into lower frequencies causing false indication of spectral lines. Care must be taken when trying to identify and deal with errors which may arise from undersampling in the time domain and aliasing in the spectral domain. The highest frequency components present in the signal must be known before a decision is made on a suitably high sampling rate.

With analogue signals it is possible to use a low-pass hardware filter before the signal is sampled in order to suppress frequency components which are too high for the sampling rate used. In addition, the signal may first be sampled at a high sampling rate and then digitally filtered to remove the higher frequency components before sampling again at a lower rate (both procedures are used with RAS analogue channels).

Compared to the sampling of analogue quantities (sampling rate, sample & hold circuit) counter/timer data acquisition channels record the time interval required for the test object to rotate through a finite angular interval as shown in Figure 6. Rather than measuring the instantaneous angular velocity,  $\omega$ , at a given point on the shaft circumference, an average velocity,  $\bar{\omega}$ , for a finite angular interval,  $\Delta\varphi$ , between adjacent measurement points is measured. This causes a damping of the measured speed amplitudes which depends on both the number of meas-

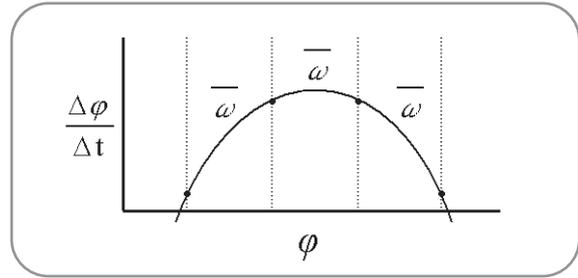


Figure 6 Angular Velocity Curve with Angle-Equidistant Data Points (•)

urement points per revolution,  $N$ , and the rotational harmonic order of interest,  $i$ .

The rotational speed fluctuation for a single order is

$$\omega(\varphi) = A \cdot \cos(i \cdot \varphi) \quad (2)$$

Since a cosine signal has its maximum value,  $A$ , at  $\varphi = 0$  and assuming that the measurement points are symmetric to  $\varphi = 0$  at  $-\Delta\varphi/2$  and  $+\Delta\varphi/2$ , then the average rotational speed in the interval  $\Delta\varphi$  is:

$$\begin{aligned} \bar{\omega}(\varphi) &= \frac{1}{\Delta\varphi} \cdot \int_{-\Delta\varphi/2}^{+\Delta\varphi/2} \omega(\varphi) \, d\varphi \\ &= \frac{1}{\Delta\varphi} \cdot \int_{-\Delta\varphi/2}^{+\Delta\varphi/2} A \cdot \cos(i\varphi) \, d\varphi \\ &= \frac{A}{i \cdot \Delta\varphi} \cdot \left[ \sin\left(i \cdot \frac{+\Delta\varphi}{2}\right) - \sin\left(i \cdot \frac{-\Delta\varphi}{2}\right) \right] \\ &= \frac{2 \cdot A}{i \cdot \Delta\varphi} \cdot \sin\left(i \cdot \frac{\Delta\varphi}{2}\right) \\ &= A \cdot \frac{N}{i\pi} \cdot \sin\left(\frac{i\pi}{N}\right) \quad (3) \end{aligned}$$



Equation (3) shows that the speed measured will always be lower than  $A$ , its maximum value.

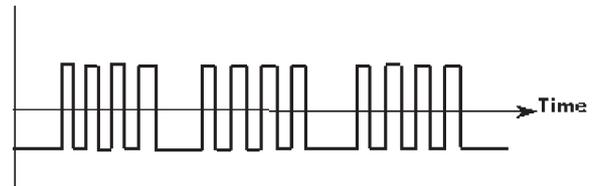
For a given order,  $i$ , amplitude damping as a percentage is given by:

$$A(i) \% = 100 \left[ 1 - \frac{N}{i\pi} \sin\left(\frac{i\pi}{N}\right) \right]$$

The ratio  $i/N = 0.5$  gives the cut-off order according to sampling theory. Accuracy is best in the limiting case of the sampling interval approaching zero, i.e. an infinite number of measurement points! Four-stroke internal combustion engines produce excitation at multiples of half engine order up to 12th order angular velocity. Using a toothed wheel with 48 teeth would mean a 0.7% degree of damping in the calculated 2nd order amplitude but a greater degree of damping (10%) for order 12. By doubling the number of teeth to 96, a lesser degree of damping results for 12th order amplitude (now 3%).

### Enhanced Resolution with Gear Tooth Sensors

Chain sprockets or gears within a gearbox possess a relatively low number of teeth. When such wheels have to be used as targets for proximity sensors, their low number of teeth prohibits FFT analysis to higher harmonics. In order to increase the amount of information per revolution obtained when using this type of wheel, a so-called 4-fold sensor has been developed. The sensor head contains four magneto-resistive elements and a permanent magnet. The sensor provides four times more information per revolution than standard speed sensors. Figure 7 shows the 4-fold pulse train.



**Figure 7** Removal of Tooth Spacing Errors

The 4-fold signal contains a repeating pattern of three short periods followed by a longer one. The three shorter periods represent the passing of a tooth; the longer period results from the next approaching tooth. A RAS software algorithm uniformly distributes the 4-fold sensor information in the revolution domain prior to time or spectral analysis. In order to obtain optimum results, the pitch of the toothed wheel should be similar to the separation of the individual magnetoresistors in the sensor head. This ensures that the longer period in the pulse trains does not dominate.

### Fast Fourier Transform Analysis

RAS software allows for both frequency and order analysis of measurement data using the Fast Fourier Transform (FFT) method. Frequency spectra require input time domain data expressed as a function of time. Order spectra require input data expressed as a function of revolutions of a reference channel. For both frequency and order analysis the time history curve is sub-divided into records (analysis intervals) of equal length. The spectral resolution is the reciprocal of the size of the analysis interval. The FFT yields  $n/2$  spectral lines for an interval size of  $n$  data points (each line has a real and an imaginary part).

A fundamental assumption of FFT analysis is that the signal contains only exact harmonics within the analysis interval. Furthermore, each harmonic component must start and stop at the same amplitude with the same slope, i.e. if the signal were immediately repeat-



ed it would be smooth and continuous. So-called leakage errors will arise if these conditions are not met. For engine tests the width of the analysis intervals is usually set to 4 revolutions of the crankshaft (2 cycles of system excitation) which ensures that only exact harmonics are contained within each interval. Care needs to be taken, however, when analysing power-train data. Transmitted engine vibrations retain their frequency but their amplitude and order will vary with transmission ratios. When order analysis is to be performed at the gearbox output, for example, exact harmonics of all signal components will not necessarily be contained within several revolutions of the output shaft. It is important that the width of the analysis intervals be chosen to reflect the signal's periodicity. Since transmission ratios often include prime numbers, the record length required for the periodic signal may be quite long. A side effect of longer analysis interval lengths is that relatively few non-overlapping spectra will result. In addition, a background speed ramp may be present in the time history data which can generate spurious harmonics in the FFT analysis. To overcome these problems shorter record lengths may be taken and a window function applied to remove sudden transients at the ends of the analysis intervals. Various window shapes may be used with the Hanning window being the most versatile [3]. Although FFT windowing reduces the leakage error it causes both damping of spectral-line amplitudes and broadening of the lines. The former may be corrected for by the RAS software based on the FFT window characteristics. The broadening of spectral lines cannot be corrected. In summary, periodicity in the analysis intervals for order analysis may be ensured by setting the interval length to a finite number of revolutions of the source of the vibration. However, frequency analysis on arbitrary time intervals requires the use of FFT windowing functions. Two applications requiring multichannel measurement and analysis will now be presented in order to illustrate the capabilities of RAS equipment.

## Timing Belt Drive System Optimisation

The primary purpose of synchronous belt drive systems is to synchronise camshafts and crankshafts. The belt system may also be used to drive integrated auxiliaries. In the 4-cylinder diesel engine shown in Figure 8 the crankshaft pulley drives the camshafts via a toothed belt. The idler on the tight side ensures that the belt wraps properly around the crankshaft's toothed wheel. The tensioner on the slack side provides tension. The fuel pump is also integrated into this drive system.

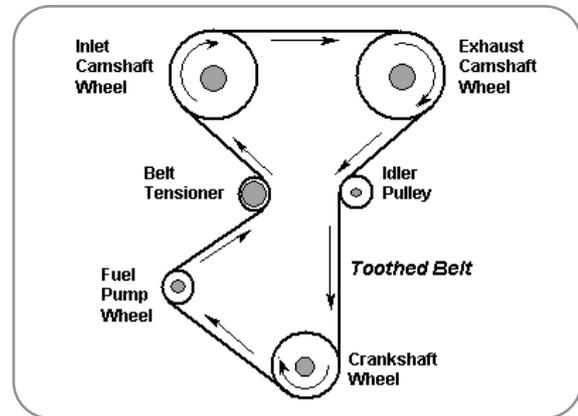
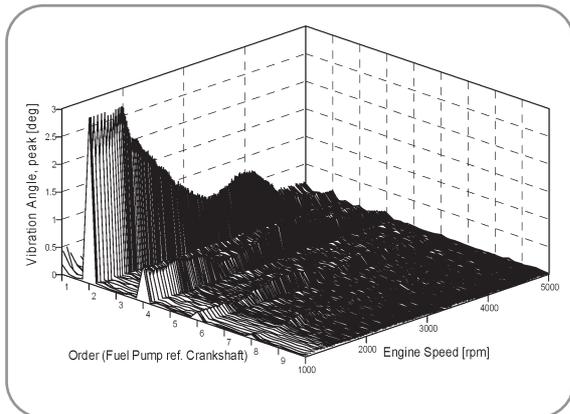


Figure 8 Synchronous Belt Drive System

The rotational speed of the crankshaft, fuel pump and camshafts was measured by scanning their toothed wheels with magnetic proximity sensors. The sensors operate up to a sensing gap of 5mm which allows for detection through the belt. In addition, displacement and force sensors provided analogue voltage signals from the tensioning system. The interaction between the components is important when optimising the system's dynamics. With the aim of minimising the dynamic load on the drive system, loads and forces in the system as well as torsional and linear vibrations need to be investigated in detail. The most versatile tool for order domain analysis today is the speed/spectrum waterfall plot where a 3-D graphical display is used to visualise the orders as

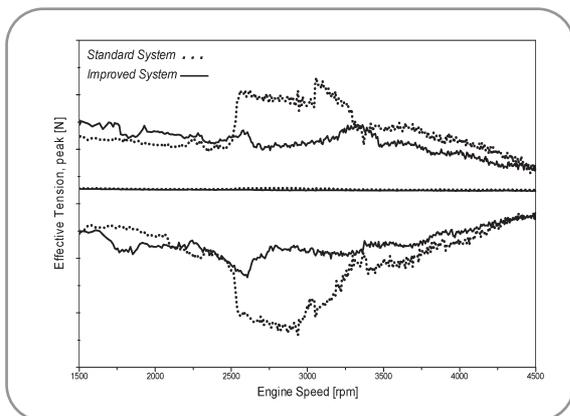


they change frequency and amplitude with speed. An example of this is shown in Figure 9 in which torsional vibration amplitudes at the fuel pump wheel referenced to the crankshaft speed display a dominant second order.



**Figure 9** Order Analysis – Waterfall Overview

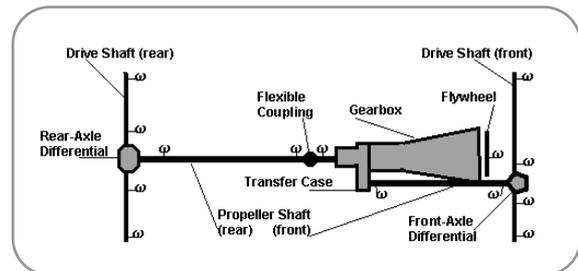
The influence of the effective tension (dynamic tight minus slack belt tension) on the life and durability of the belt is significant. Figure 10 shows the effective tension on the belt both before and after improvements were made to the system. Lower torsional vibration and belt tension fluctuation were present in the speed range around 3000rpm following improvements to the system.



**Figure 10** Belt tension versus Speed

**Powertrain of a 4WD-Vehicle**

One of the most comprehensive investigations made with RAS equipment to date involves the instrumentation of a four-wheel-drive vehicle test rig. The RAS system used was fitted with a total of 16 speed and 40 analogue channels as well as inputs for CAN-bus and triggering/pre-triggering signals. The flywheel teeth and synchronisation signals were used both to trigger data acquisition and provide a reference for the engine rotational position. The angular velocity of several gears within the gearbox was measured. Special toothed wheels were added to the shafts and scanned either with standard magneto-resistive sensors or with 4-fold sensors for improved resolution. The vibration of the gearbox casing was measured with ICP accelerometers. Additionally, force, temperature and displacement sensors were fitted. The instrumentation is outlined in Figure 11 where angular velocity measurement points are indicated by the symbol  $\omega$ .



**Figure 11** Angular Velocity Measurement Positions

Some objectives of the tests conducted were:

- ▶ Load change problems (vibration behaviour when switching from drive to coast)
- ▶ Transmission accuracy of differential gears
- ▶ Improving driving comfort by introducing vibration absorbers and a dual-mass flywheel
- ▶ Noise emission due to gearbox rattling



These types of multichannel investigations help in the understanding and interpretation of vehicle dynamics and provide ideal input data for validating computer simulations used in predicting and visualising engine and powertrain NVH behaviour.

## Conclusions and Outlook

This paper reviewed and discussed several aspects of multichannel measurement and analysis of torsional vibration and related data. Shortening development cycles while simultaneously increasing the volume and diversity of noise and vibration testing constitute conflicting requirements in modern NVH engineering. This emphasises the pivotal role of state-of-the-art testing tools.

Both time constraints and physical limitations in accessing measurement positions mean that test engineers have little chance in practice of acquiring all-embracing experimental data. Information flow between testing and computational departments will therefore have to be improved and optimised. Only an integrated approach which combines mathematical modelling and experimental testing can help reduce the number of prototypes and further shorten development cycles [7].

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